

# Experimental scheme for quantum teleportation of a single-photon packet

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## Abstract

Both complete protocol and optical setup for experimental realization of quantum teleportation of unknown single-photon wave packet are proposed.

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Quantum mechanics prohibits cloning (copying) an unknown quantum state (no cloning theorem [1]). Is it possible to transmit to a distant user a previously unknown quantum state without sending that state itself? Strictly speaking, any measurement aimed at obtaining classical information to be sent to another observer changes the state itself without providing complete information about it. Production of a large number of identical copies which can be measured many times to obtain complete information on their common quantum state is prohibited by the no-cloning theorem. Thus, it is impossible to transmit information on a quantum state employing only a classical communication channel.

Quantum teleportation lifts this restriction if a quantum communication channel is used in addition to the classical one. The idea of quantum teleportation for the case of discrete quantum states (e.g. a spin-1/2 particle in an unknown state) was first proposed in Ref.[3]. A quantum channel is realized through the non-local EPR-correlations [2,3]<sup>1</sup>. An EPR-pair is a pair of particles described by an entangled state. Entanglement constitutes a special kind of quantum correlations which do not have any classical analogies.

The quantum teleportation protocol described in Ref.[4] has the following form. To teleport an unknown quantum state from user *A* to a distant user *B*, user *A* generates an EPR-pair. One of the particles of that EPR-pair remains with the user *A* while the second one is sent to the distant user *B*. User *A* performs a joint measurement over the particle in the unknown state to be teleported and his particle from the EPR pair thus obtaining classical information. Because of the non-local correlations inherent in the EPR-pair, the measurement outcome uniquely determines the resulting state of the second particle in the EPR-pair sent to user *B*. The state of the second particle coincides with the unknown state to within a unitary rotation. Classical information obtained in the measurement is sent by user *A* to *B* and is used by the latter to determine which unitary transformation should be performed to obtain a new state identical to the original unknown state. In the course of teleportation user *A* obtains no information on the teleported unknown state.

Quantum teleportation has recently been demonstrated experimentally for a photon in an unknown polarization state [5,6].

The problem of teleportation of the wave function of a particle in the one-dimensional case where position and momentum are the two continuous dynamic variables was investigated in Ref.[7] using the wave function proposed in Ref.[2] to describe an EPR pair. Studied in a recent work [8] was the quantum teleportation of a quantum state described by the dynamical variables (the unknown state in Ref.[8] corresponds to a single-mode photon state) for the case of non-ideal EPR-correlations. A quadrature-squeezed state was used as an EPR-state while the measurement procedure actually corresponded to the homodyne detection.

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<sup>1</sup>This term stems from the well-known Einstein-Podolsky-Rosen effect [2]

Proposed in the present paper is a new scheme (a complete protocol and its experimental realization) for the teleportation of a multi-mode state, i.e. a single-photon wave packet, employing an EPR-pair in an entangled state with respect to the energy–time variables.

To simplify further formulas, we shall assume that the packet polarization state is known. The discussion below is also applicable to the case of unknown polarization which can be accounted for introducing an additional subscript. The state of a single-photon wave packet can be written as (see, e.g. Ref.[9])

$$|1\rangle_3 = \int_0^\infty d\omega f(\omega) \hat{a}^+(\omega) |0\rangle = \int_0^\infty d\omega f(\omega) |\omega\rangle_3, \quad (1)$$

$$[\hat{a}(\omega), \hat{a}^+(\omega')] = I\delta(\omega - \omega'), \quad \int_0^\infty |f(\omega)|^2 d\omega = 1,$$

where  $\hat{a}^+(\omega)$ ,  $\hat{a}(\omega)$  are the creation and annihilation operators of a single-mode Fock state  $|\omega\rangle_3$ ,  $|0\rangle$  is the vacuum state,  $f(\omega)$  is the packet amplitude Subscript 3 labels the channel number (see Fig.1). The density matrix at an arbitrary time is

$$\rho(3) = \left( \int_0^\infty d\omega e^{-i\omega t} f(\omega) |\omega\rangle_3 \right) \left( \int_0^\infty d\omega' \langle \omega | e^{i\omega' t} f^*(\omega') \right) \quad (2)$$

In our case the state of an EPR-pair can be written as (these photon EPR-pairs are produced in the parametric energy down-conversion processes [10])

$$|\psi_{EPR}\rangle_{1,2} = \int_0^\infty d\omega |\omega\rangle_1 \otimes |\Omega - \omega\rangle_2, \quad \rho_{EPR}(1,2) = |\psi_{EPR}\rangle_{1,2} \langle \psi_{EPR}|, \quad (3)$$

where  $\Omega$  is the pumping frequency and 1, 2 are the channel numbers (Fig.1). The normalization of the state (3) is insignificant for further analysis.

According to the general scheme [11–13], quantum mechanical measurements are described by positive operators realizing the identity resolution. Measurements of the variables corresponding to self-adjoint operators are associated with the orthogonal resolutions. Parameters (such as time and rotation angle) are not associated with any self-adjoint operators so that the corresponding measurements are described by the non-orthogonal identity resolution [11–13].

In the present paper the basic idea of proposed teleportation scheme consists in the usage of a joint (entangled) time–energy measurement performed on a pair of photons one of which belongs to the EPR-pair and the second photon in the unknown state to be teleported. The measurement is given by the non-orthogonal identity resolution [14]

$$\int \int M(dt d\Omega_+) = \int \int R^+ R (dt d\Omega_+) = I, \quad (4)$$

where  $R$  is the “reduction” operator,  $M(dt d\Omega_+)$  describes a quantum operation and is a positive operator valued measure, POVM, (the details can be found, e.g. in Refs.[15,16]); we have [14]

$$M(dt d\Omega_+) = \quad (5)$$

$$\left( \int d\omega_- e^{i\omega_- t} |\omega_+ + \omega_-\rangle_1 \otimes |\omega_+ - \omega_-\rangle_3 \right) \left( \int d\omega'_- e^{-i\omega'_- t} \langle \omega_+ - \omega'_- | \otimes \langle \omega_+ + \omega'_- | \right) \frac{dt d\omega_+}{2\pi},$$

$\omega_\pm = \frac{\Omega_\pm}{2}$ . The integration covers the frequency ranges corresponding to positive arguments of the Fock states. It should be emphasized that the frequency  $\omega_+$  is common to bra- and ket-states.

According to the general concepts of the quantum measurement theory [10–13,15,16], application of a quantum operation (measurement) to a system described by the density matrix  $\rho$  transforms it to a new state

$$\rho \rightarrow \frac{R_i \rho R_i^+}{\text{Tr}\{R_i \rho R_i^+\}}.$$

The probability of the  $i$ -th outcome is given by the formula  $\text{Pr} = \text{Tr}\{R_i \rho R_i^+\}$ , where  $E_i = R_i^+ R_i$  is the POVM element.

In our case, after the measurement performed by user  $A$ , the state of the second photon from the EPR-pair observed by user  $B$  is given by the density matrix

$$\tilde{\rho}(2) = \frac{\text{Tr}_{1,3}\{\rho_{EPR}(1,2) \otimes \rho(3)M(dtd\Omega_+)\}}{\text{Pr}\{dtd\Omega_+\}}, \quad (6)$$

$$\text{Pr}\{dtd\Omega_+\} = \text{Tr}_{1,2,3}\{\rho_{EPR}(1,2) \otimes \rho(3)M(dtd\Omega_+)\}, \quad (7)$$

$$\tilde{\rho}(2) = \left( \int d\omega e^{-i(\Omega_+/2+\omega)t} f(\Omega_+ - \omega) |\Omega - \omega\rangle_2 \right) \left( \int d\omega' e^{i(\Omega_+/2+\omega')t} {}_2\langle \Omega - \omega' | f^*(\Omega_+ - \omega') \right) \quad (8)$$

Formally, the measurement (4) corresponds to the situation where the measurement moment  $t$  and frequency  $\Omega_+$  are chosen by the experimentator, and the positive result probability is given by Eq. (7). The teleportation will be ideal if the chosen  $\Omega_+ = \Omega$  (detector registration frequency  $\omega_+$  coincides with the pumping frequency  $\Omega$ ). In that case it follows from Eq. (6) that the state  $\tilde{\rho}(2)$  coincides with  $\rho(3)$  to within a phase factor which can be eliminated by user  $B$  if user  $A$  sends to him the registration time  $t$  via a classical channel. Note that the registration time  $t$  does not depend on unknown state  $\rho(3)$  if  $\Omega_+ = \Omega$ .

Physically, the measurement (4) can be understood in the following way. User  $A$  has a continuum of detectors “tuned” to the frequencies in the  $(0, \infty)$  range, and each of them can fire at an arbitrary time  $t$ , formally in the infinite  $(-\infty, \infty)$  interval. The probability for the detector tuned to frequency  $\Omega_+$  to fire at time  $t$  is given by Eq. (7). The firing probability does not depend on time only for the detector tuned to frequency  $\Omega$ . The teleportation will only be ideal if the detector tuned to  $\Omega$  fires. In that case it follows from Eq. (7) that the firing probability does not depend on the unknown input state. User  $A$  does not obtain any information on the teleported state.

Since the measurements can be performed at spatially separated points, all the times in the above formulas should be understood as the reduced time-of-flight corrected times ( $t \rightarrow t - x/c$ ). It will be seen below that this point is insignificant in the case of ideal teleportation.

The major problem in realizing the single-photon packet teleportation is the implementation of the measurement given by Eq. (4). The measurement (4) on a pair of photons is an intermediate case between the time and energy measurements; its experimental realization is described below. The idea is to convert a photon pair into a single photon which then is measured by narrow-band photodetector. The latter can be easily realized experimentally.

The experimental setup is presented in Fig.1. The first non-linear crystal with the second-order susceptibility  $\chi$  and a narrow-band filter tuned to the frequency  $\Omega$  serve to generate the EPR-pair in channels 1 and 2. The channel 3 is used to feed the unknown single-photon packet. The latter can be prepared by exciting a two-level system with a  $\pi$ -pulse a long time ago. The second non-linear crystal, the narrow-band filter behind it tuned to frequency  $\Omega$ , and then a standard photodetector realize the measurement (4). The teleported state arises in channel 2.

Consider step by step the input state evolution in the optical scheme. After the first narrow-band filter in front of the first non-linear crystal the state is described by a monochromatic state with the density matrix

$$\rho_{in}(in) = |\Omega\rangle_{in} \langle \Omega|, \quad (9)$$

which can be obtained by cutting a narrow band by the first filter from an auxiliary single-photon packet fed into the input channel *in* (Fig.1). The photon-photon interaction in the nonlinear crystal is described in the interaction representation by the following Hamiltonian (details can be found in Refs. [17,18])

$$H_1(t) = \chi \int d\mathbf{x} E_{in}^{(+)}(\mathbf{x}, t) E_1^{(-)}(\mathbf{x}, t) E_2^{(-)}(\mathbf{x}, t) + h.c., \quad (10)$$

all the insignificant constants are assumed to be included in the definition of  $\chi$ , which as usually [17,18] will be assumed to be frequency-independent. It is convenient to present the electric field operators in the form [9]

$$E_i^{(-)}(\mathbf{x}, t) = \frac{1}{\sqrt{2\pi}} \int_0^\infty d\omega e^{i(\omega t - \mathbf{k}\mathbf{x})} \hat{a}^+(\omega) |0\rangle_i = \frac{1}{\sqrt{2\pi}} \int_0^\infty d\omega e^{i(\omega t - \mathbf{k}\mathbf{x})} |\omega\rangle_i, \quad (11)$$

where  $i$  is the channel number. Analogously for  $E_i^{(+)}(\mathbf{x}, t)$ . Taking into account Eq. (11) we have

$$H_1(t) = \frac{\chi}{(2\pi)^{3/2}} \int \int \int d\omega_1 d\omega_2 d\omega_{in} e^{it(\omega_1 + \omega_2 - \omega_{in})} |\omega_1\rangle_1 \otimes |\omega_2\rangle_2 \langle \omega_{in}| \int_{vol} d\mathbf{x} e^{-i\mathbf{x}(\mathbf{k}_1 + \mathbf{k}_2 - \mathbf{k}_{in})} + h.c. \quad (12)$$

In the second integral the integration is performed throughout the entire crystal yielding the  $\delta$ -symbol with respect to momentum, and results in the phase synchronism conditions [18] ( $\mathbf{k}_1 + \mathbf{k}_2 = \mathbf{k}_{in}$ ), which will be assumed satisfied (below it means that  $\mathbf{k}_2 \parallel \mathbf{k}_3$ , Fig.1). In the subsequent formulas the quantity  $\chi$  is understood as the renormalized constant corrected for the additional factors arising from the second integral. The first-order susceptibility which is always present can be neglected in our analysis since the corresponding terms in the Hamiltonian do not contribute to the *out* channel.

The state after the first crystal in channels 1 and 2 is described by the density matrix

$$\rho_{EPR}(1, 2) = S(t) \rho_{in}(in) S^{-1}(t), \quad (13)$$

where  $S(t)$  is the  $S$ -matrix

$$S(t) = e^{i \int_{-\infty}^t H_1(t') dt'} = 1 + S^{(1)} + S^{(2)} + \dots \quad (14)$$

In the first order with respect to  $\chi$  one has

$$S^{(1)} = i\chi \int \int d\omega_1 d\omega_{in} |\omega_1\rangle_1 \otimes |\omega_{in} - \omega_1\rangle_2 \langle \omega_{in}| + h.c. \quad (15)$$

The upper integration limit in the exponent in  $S$  can be replaced by  $\infty$  which is physically actually related to the fact that the input state is a monochromatic one (roughly speaking infinitely extended in time) and the teleportation process is formally stationary. To within an unimportant normalization constant, the state in channels 1 and 2 is described by the density matrix

$$\rho_{EPR}(1, 2) = \chi^2 \left( \int_0^\infty d\omega |\omega\rangle_1 \otimes |\Omega - \omega\rangle_2 \right) \left( \int_0^\infty d\omega' \langle \omega'| \otimes \langle \Omega - \omega'| \right). \quad (16)$$

Measurement by a photodetector which has a narrow-band filter tuned to the frequency  $\Omega$  installed in front of it is formally described by the projector  $P(\Omega) = |\Omega\rangle_{out} \langle \Omega|$ .

The teleported state in channel 2 after the registration by the photodetector is described by the density matrix (again to within the normalization constant)

$$\tilde{\rho}(2) = \text{Tr}_{out} \left\{ S(t) \rho_{in}(in) \otimes \rho(3) S^{-1}(t) P(\Omega) \right\}, \quad (17)$$

where  $S(t)$  is now the full  $S$ -matrix of the entire optical scheme,

$$S(t) = e^{i \int_{-\infty}^t [H_1(t') + H_2(t')] dt'} = 1 + S^{(1)} + S^{(2)} + \dots, \quad (18)$$

where  $H(t)_2$  is the second non-linear crystal Hamiltonian which coincides to within the subscripts interchange with  $H(t)_1$  in Eq. (12). Contributing to the teleportation processes are the  $S$ -matrix terms of the form

$$S^{(2)} \propto \quad (19)$$

$$\chi^2 \left( \int \int d\omega_1 d\omega_{in} |\omega_1\rangle_1 \otimes |\omega_{in} - \omega_1\rangle_2 \langle \omega_{in}| \right) \left( \int \int d\omega'_1 d\omega_{out} |\omega_{out}\rangle_{out} \langle \omega'_1| \otimes \langle \omega_{out} - \omega'_1| \right).$$

Taking into account Eq. (19), the density matrix in channel 2 to within the normalization constant coincides with the initial density matrix of the unknown wave packet

$$\tilde{\rho}(2) = \chi^4 \left( \int_0^\infty d\omega f(\omega) |\omega\rangle_3 \right) \left( \int_0^\infty d\omega'_3 \langle \omega | f^*(\omega') \right) \quad (20)$$

It also follows from Eq. (17) that the detection probability in the output channel (*out*) does not depend on the unknown state and is proportional to

$$\text{Pr} = \text{Tr}_{2,out} \left\{ S(t) \rho_{in} (in) \otimes \rho(3) S^{-1}(t) P(\Omega) \right\} \propto \chi^4. \quad (21)$$

In the outlined scheme the classical channel is used to inform the distant user of the fact that a photodetector fired, and in that case the teleportation is assumed to be successful. The probability (efficiency) of the teleportation process is small to the measure of  $\chi^4$ . The fraction of false photodetection firings when a wrong state will be teleported due to the terms of higher orders in  $\chi$  in the  $S$ -matrix has an additional smallness in  $\chi^2$ . Note that formally the teleportation process is stationary (requires infinite time), since a monochromatic state should be prepared. In that case the probability of the photodetector firing in the *out* channel does not depend on time and does not depend on the input state. In that case user *A* obviously acquires zero information on the teleported state.

Of course, quantum teleportation does not allow information transmission faster than the speed of light. In the present scheme an intuitive and qualitative explanation is as follows. Since the input state is monochromatic and always intuitively assumed to be non-localized (infinitely extended), the latter means that the field is “prepared in advance” throughout the entire space, including the locations of both distant users *A* and *B*. The measurement performed by user *A* transforms the entire system to a new state—reduces the state vector “immediately” and “everywhere” for the entire system. This assumption is usually considered as counterintuitive. However, these “immediately” and “everywhere” do not result in faster than light communication. To transmit classical information from *A* to *B* with the help of a teleported state, one requires a classical channel from *A* to *B* to tell that the detector fired and the teleportation was successfully completed. Classical communication channel assumes sending a classical object from *A* to *B* whose velocity cannot exceed that of light. The problem of the field “prepared in advance” everywhere is closely related to the photon localizability (to be more precise, non-localizability) (e.g. see Refs.[19–23]). As far as I know this problem has not yet been discussed in detail in the context of quantum teleportation.

It should be noted that the teleportation process can be reformulated in the Feynman diagram language, the averaging being performed over the stationary state  $|\Omega\rangle \otimes \int_0^\infty d\omega f(\omega) |\omega\rangle$  corresponding to the input monochromatic state and the single-photon state. In the case of ideal teleportation this state is also the output state. In that sense the process is stationary and the averaging is performed over the stationary state which should not necessarily be the

ground state. In that case the diagrammatic technique is developed in the same way as in the Keldysh method [24].

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